MATH 220 (CALCULUS 2) MOCK FINAL EXAM, SPRING 2007

This contains about 20% more questions than the actual exam will, to cover a fuller array of topics.

(1) Compute the following limits, derivatives and integrals
  (a) $\lim_{x\to\infty} e^{-x} \ln x$
  (b) $y = (1 + x^2)\tan^{-1} x$, $dy/dx =$
  (c) $f(x) = \tanh(\ln x)$, $f'(x) =$
  (d) $\displaystyle\int \frac{3x+2}{x^2+6x+8}\,dx$
  (e) $\displaystyle\int x \sin 4x\,dx$
  (f) $\displaystyle\int \sin^2 x \cos^3 x\,dx$
  (g) $\displaystyle\int \frac{dx}{(4-x^2)^{3/2}}$

(2) Evaluate $\displaystyle\int \tan^4(2x)\,dx$ using the reduction formula

$$\int \tan^n(ax)\,dx = \frac{\tan^{n-1}(ax)}{a(n-1)} - \int \tan^{n-2}(ax)\,dx.$$

(3) Write an expression in $\sum$ notation for the approximation of $\displaystyle\int_1^3 \sqrt{\sin x}\,dx$ by the Midpoint Rule with 4 intervals. Then evaluate this approximation.

(4) (a) Explain why $\displaystyle\int_{-8}^{27} \frac{dx}{x^{1/3}}$ is an improper integral, and either evaluate it or show that it is divergent.
  (b) Evaluate $\displaystyle\int_2^{\infty} e^{-2x}\,dx$.

(5) Give the form of the partial fractions expansion for $\displaystyle\frac{x^2+2x+4}{(x^2+1)^2(x+2)^3}$.
There is no need to evaluate the constants.

(6) Sketch the region bounded by the curves $y = 2x^2$ and $y = x^3$, and compute the area of this region.

(7) Consider the region in the first quadrant bounded by the y-axis, the curve $y = \sqrt{x}$, and the line $y = 4$.
  (a) Compute the volume of the region produced by rotating this region about the x-axis.
  (b) Compute the volume of the region produced by rotating this region about the y-axis.

(8) If the velocity of a car moving away from a stop sign t seconds after starting to move is $v = 5t - t^2$ meters/second, compute its average velocity over the first two seconds.

(9) Find the general solution of the differential equation

$$\frac{dy}{dx} = x^2 e^y$$

and use this to find the particular solution that passes through the point $(4, 2)$.

(10) One thousand of a new species of fire ant were accidentally introduced into Charleston 18 months ago and with no natural predators, the population is growing according to the law of exponential growth. The population is now 30,000: estimate when the population will reach one billion.

(11) Solve the linear differential equation

$$x^2 \frac{dy}{dx} = \cos x - 2xy$$

(12) Sketch the parametric curve $x = \cos^2 t$, $y = \cos t$, $0 \leq t \leq 2\pi$. Indicate the points corresponding to $t = 0, \pi/4, \pi/2, 3\pi/4, \pi \ldots 2\pi$, and the direction of motion along the curve.

Then compute a formula for the slope of the curve, and use this to find where the curve has a vertical tangent: check against your graph!

(13) Write an integral expression for the length of the parametric curve

$$x = e^{2t} - 2t, \quad y = 4e^t, \quad 0 \leq t \leq 2,$$

simplify as much as possible, and compute the length of the curve.

(14) Sketch the curve $r = \sqrt{\theta}$ in the x-y plane for $0 \leq \theta \leq 2\pi$.

(15) Compute the area inside one loop of the polar curve $r = \sin 3\theta$.

(16) Find a direct formula "a_n" and a recursive description for a sequence that starts

$$1, -\frac{2}{3}, \frac{6}{9}, -\frac{24}{27}, \frac{120}{81} \cdots$$

(17) Write out the first three terms of each sequence below, and then either find the value of the limit of the sequence if it exists, or explain why it does not exist.

(a) $a_n = \dfrac{n + \sin n}{2n + 4}$

(b) $a_n = \dfrac{(-1)^n}{1 + 2/n}$

(18) (a) Write out the first four terms of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot n!}$ and compute the third and fourth partial sums.

(b) Explain why this series converges, and determine how many terms would you have to sum in order to approximate the value of the infinite sum within 0.01.

(19) Derive the quadratic Taylor polynomial approximation with center $a = 0$ for

$$f(x) = \sqrt[3]{x + 8}$$

and use it to compute an approximation of $\sqrt[3]{9}$.

(20) Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(1 + n^3)x^n}{n!}$.

(a) Write out the first three terms of the series.
(b) Compute the radius of convergence of the series.

(21) No elementary anti-derivative exists for the function $\sin(x^2)$ but we can approach its integral as follows.

(a) Using known power series, derive a power series for $f(x) = \sin(x^2)$. Give the first three non-zero terms as well as describing the whole series using summation (Σ) notation.
(b) Use the above power series for $\sin(x^2)$ to derive a power series for $\int \sin(x^2)\, dx$. Give the first three non-zero terms after the constant, as well as the general term or an expression in sigma notation.
(c) Explain why both series above converge for all x.