MATH 545 COMPUTATIONAL PROJECT 3: BOUNDARY VALUE PROBLEMS AND EIGENVALUES

DUE FRIDAY APRIL 20, WITH POSSIBILITY OF REVISIONS

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1. Nonlinear boundary value problems by shooting

Solve the nonlinear boundary value problem

$$\frac{d^2y}{dx^2} = 2y^3, \qquad y(1) = 1/4, \quad y(2) = 1/5. \tag{1}$$

accurate to an estimated two decimal places. Do this by shooting, using the Matlab function `fzero` to solve the necessary nonlinear equation $F(\gamma) = 0$. To build the Matlab function `F(gamma)`, rewrite the above ODE as a system of two first order equations, and use any of ODE IVP solver that can handle systems, either ones of mine or standard Matlab ones.

Present the results graphically, and then check accuracy given that the exact solution is $y(x) = 1/(x + 3)$.

2. ODE eigenvalue problems and matrix eigenvalue problems

Compute numerically the lowest eigenvalue and corresponding eigenfunction for

$$-\frac{d^2y}{dx^2} + (2 + \sin \pi x)y = \lambda y, \qquad y(0) = 0, \quad y(1) = 0. \tag{2}$$

Do this by discretizing in the standard way to a matrix eigenvalue problem, and solving that for $n = 10$, $n = 20$, and $n = 40$ intervals.

First do this using the Matlab function `eig`. Compare graphically to verify that the computed eigenfunctions and eigenvalues appear to be converging as n increases.

Then do it again by writing your own Matlab function based on the inverse power method.

The form of project reports and submissions

The submission should consist of a report on paper (and preferably also in electronic form like a TeX source file and PDF or a Word file) plus an online folder containing supporting files such as Matlab functions and scripts, output diaries and graph files in some standard format like JPEG or PDF. The files should be gathered in a folder; that folder should have a name including your name and identifying project that it is for (e.g LeMesurier-Project3). Files within that folder need not have your name as part of their names, but your name, the date and a title should appear in within all documents such as the final report and Matlab function and script files.

If new versions are submitted, distinguish them with the date or a version letter (e.g. folder LeMesurier-Project3-0409, or LeMesurier-Project3-0409b for a second version later on April 9. Other files need not have your name as part of their name; particularly not Matlab files: simple names like 'powermethod.m' are better.

The report should explain all mathematical work that you have done in the project, such as algorithms devised and/or implemented by you, summaries of main computational results with numerical values and graphs as appropriate, and comments on what these results show about the accuracy, reliability and efficiency of the numerical methods used. Details of basic algorithms covered in class are not needed; for example, there is no need to describe the inverse power method, only to explain what you do with it in your project work.

I highly encourage submitting drafts and partial work before the due date, with questions or just for feedback, and to email me questions, perhaps accompanied by any Matlab files that the questions are about. Do this as much as possible in electronic form. When you drop off work at any stage, send email letting me know, and include everything you have on the project so far, including files already dropped off earlier.

I also strongly suggest that you discuss your plans for the algorithms necessary with me before getting too far into the Matlab programming,

Note that some of the algorithms used will be ones already implemented in Matlab functions such as `ode45`, `fzero`, `eig` or my ODE solver functions, and these do not need to be discussed in your reports.