A Faster Backward Stable Bidiagonalization Procedure

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ABSTRACT

A new bidiagonalization method is proposed for $X \in \mathbf{R}^{m\times n}$. For $m \geq n$, it decomposes X into the product

$$X = UBV^T$$

where $U \in \mathbf{R}^{m\times n}$, $V \in \mathbf{R}^{n\times n}$ are orthogonal and B is bidiagonal. The matrix V is computed as a product of Householder transformations and the matrices U and B are constructed using a Lanczos based recurrence. If U is desired from the computation, this saves as much as 25% of the flops over the Golub–Kahan procedure [SIAM J. Num. Anal. Series B, 2:205–224,1965] or similar procedures. It has the disadvantage that, in floating point arithmetic, U may be far from orthogonal.

Fortunately, the singular values of B are those of X perturbed by no more than $f(m,n)\varepsilon_M\|X\|_2$ where $f(m,n)$ is a modestly growing function and ε_M is the machine unit. Moreover, the application of any backward stable singular value decomposition procedure to B recovers the left singular vectors associated with the leading singular values of X to near orthogonality. Thus this leads to faster procedures for recovering the leading singular values and associated singular vectors of X.