The Truncation Method for Approximating Stationary Probabilities

G. W. Stewart

Department of Computer Science
Institute for Advanced Computer Studies
University of Maryland
College Park, MD

ABSTRACT

Let Q be the generator for an infinite, discrete-time, discrete-space Markov chain with stationary probability vector $\boldsymbol{\pi}^{\mathrm{T}}$. Since it is not possible to compute all the components of $\boldsymbol{\pi}^{\mathrm{T}}$, it is customary to partition the equation $\boldsymbol{\pi}^{\mathrm{T}}Q = 0$ in the form

$$(\boldsymbol{\pi}_1^{\mathrm{T}} \ \ \boldsymbol{\pi}_2^{\mathrm{T}}) \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} = 0.$$

and compute approximations to $\boldsymbol{\pi}_1$ from Q_{11}. Traditionally, one adjusts Q_{11} so that it is a generator and takes the stationary vector of that matrix as the approximation. In this talk we examine the consequences of leaving Q_{11} unaltered and taking the positive eigenvector of Q_{11} as the approximation. Specific bounds are developed that show that the accuracy of the approximation depends on the size of $\boldsymbol{\pi}_2^{\mathrm{T}}$.