Existence and Stability Criteria for Ring Neural Networks of Spiking Neurons

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Introduction

Traditionally, phase resetting is a function of phase only, as can be seen in Fig. 1 (a reciprocally coupled case). The stable firing patterns of small neural networks can be solved analytically if the following assumptions are made:

1) all component neurons in the circuit are endogenous bursters.
2) there are no synaptic delays.
3) each neuron receives one perturbation (synaptic input) per cycle.
4) this perturbation takes the same form in the closed loop circuit as in an open loop circuit composed only of a presynaptic neuron driving a postsynaptic one.
5) each neuron returns to its unperturbed limit cycle before the next input is received.

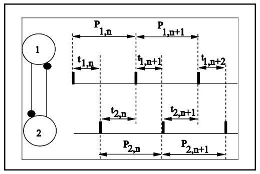


Figure 1. Two entrained neurons that are reciprocally coupled.

However, for the experimentalist, there is no such thing as a delta function and a more realistic representation of the reciprocally coupled case would have rectangles inserted instead of the delta spikes in Fig. 1, where the amplitude and duration of the rectangular perturbing pulse may be varied as independent variables.

Problem

There are several problems with this representation however. One of these problems is due to the fact that any change in the shape of the stimulus will result in a different PRC corresponding to that stimulus. In addition to the computational difficulties, experimentally, this means that PRCs must be extracted for every possible stimulus.

Solution

This is what we did using a Morris-Lecar model. The idea was that scaling laws might be extracted from these simulations. If the scaling laws are known about the system, then potentially only a single PRC is necessary to re-create the others and the recursion formulas for phase resetting can be expanded to include and account for changes in the amplitude and the duration of rectangular perturbing pulses.

Data

The primary portion of our data is composed of the **200,000 PRCs** that we generated with varying values of:

- **I_0** (what we are calling "zero" influx of current)
- **ϕ** (time constant)
- **Delta I_0** (amplitude of perturbing pulse)
- **Duration** (duration of perturbing pulse)
- **Voltage Threshold** (the decider of phase zero)

PRC Scaling

The existence and stability of of 1:1 phase-locked modes in neural networks is hinged upon the smoothness in changes between the different PRCs caused by possible perturbations. In a real system, there will always be some noise or unpredictable changes in the anatomy of the perturbing pulses received by a neuron. If a PRC is scalable, this means that, in theory, small changes in the perturbations will not cause wildly un-similar PRCs and that the differences between them are predictable. In other words, the stability of entrained modes in neural networks depends on how the PRCs scale with changes in input.

Varying Amplitude

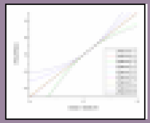


Figure 9. Δt = 11.3386ms.

Figure 10. Δt = 56.69ms

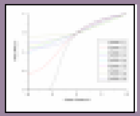


Figure 11. Δt = 22.68ms

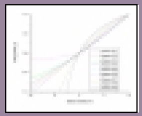


Figure 12. Δt = 16.3386ms.

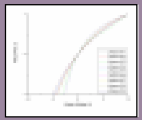


Figure 13. Δt = 113.39ms

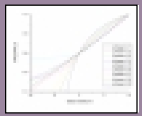
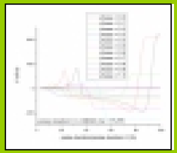


Figure 14. Δt = 34.02ms

Varying Duration

Figure 2. I_0 = 0.081, phi = 0.01, Voltage Threshold = 0.2, $\Delta I = (0.014)I_0$



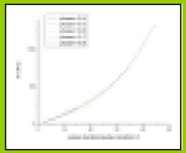
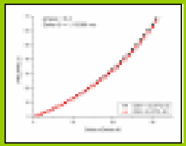


Figure 3. I_0 = 0.081, phi = 0.01, Voltage Threshold = 0.2, $\Delta I = (0.014)I_0$

Figure 4. I_0 = 0.081, phi = 0.02, Voltage Threshold = 0.0, P_0 = 113.4 ms.



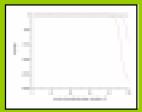


Figure 5. I_0 = 0.081, phi = 0.02, Voltage Threshold = 0.0.

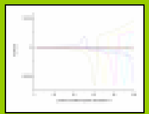


Figure 6. I_0 = 0.081, phi = 0.02, Voltage Threshold = 0.0.

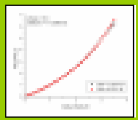


Figure 7. I_0 = 0.081, phi = 0.02, Voltage Threshold = 0.0.

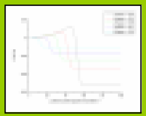


Figure 8. I_0 = 0.081, phi = 0.02, Voltage Threshold = 0.0.

Stability

Referring to Fig 1. For the case where $F = F(\varphi)$, the recursion formulas can be derived:

$$t_1[n] + t_2[n] = P_{1,0}(1 + F_1(t_1[n]/P_{1,0})$$
$$t_2[n] + t_1[n+1] = P_{2,0}(1 + F_2(t_2[n]/P_{2,0})$$

$$\delta\varphi_1[n+1] = \lambda\delta\varphi_1[n] \text{ Where: } \lambda = (1 - m_1)(1 - m_2)$$

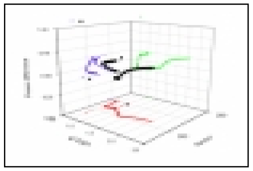
The system is stable if: $|\lambda| < 1$, and unstable if $|\lambda| > 1$. However, if Phase Resetting becomes a function of pulse duration and amplitude as well ($F = F(\varphi, \tau, I)$) then the recursion becomes:

$$\delta\varphi_1[n+1] = \lambda\delta\varphi_1[n] - (\delta\tau/\tau_0)(1 - m_2) F_1(\varphi^*_1, \tau_0, I_0) - (\delta I/I_0)(1 - m_2) F_1(\varphi^*_1, \tau_0, I_0)$$

Where "τ" is pulse duration and "I" is pulse amplitude.

Two Coupled Neurons

Figure 15. Two entrained neurons with a varying amplitude from neuron "b" to neuron "a"



Summary

In a local environment, varying the perturbing pulse amplitude and duration gives optimistic results supporting the idea that these PRCs scale in a well behaved way. The data concerning the varying of amplitude show for some phases that the PRCs scales linearly. Interestingly, varying pulse duration suggests that the PRCs scale quadraticly within a certain region after a region of linear scaling for small changes in duration.

Acknowledgements

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References

1 Oprisan, S. A., & Canavier, C. C. (2000a). Phase response curve via multiple time scales analysis of the limit cycle behavior of type I and type II excitability. Biophys. J., 78, 218A.

2 Oprisan, S. A., & Canavier, C. C. (2000b). Analysis of central pattern generator circuits using phase resetting. Soc. Neurosci. Abstr., 26, 2000.

3 Oprisan, S. A., & Canavier, C. C. (2001). The structure of instantaneous phase resetting in a neural oscillator. InterJournal of Complex Systems, manuscript 386. Available on-line at: www.interjournal.org/.

4 Canavier, C. C., Baxter, D. A., Clark, J. W., & Byrne, J. H. (1999). Control of multistability in ring circuits of oscillators, Biol. Cybernetics, 80, 87-102.

5 Canavier, C. C., Butera, R. J., Dror, R. O., Baxter, D. A., Clark, J. W., & Byrne, J. H. (1997). Phase response characteristics of model neurons determine which patterns are expressed in a ring circuit model of gait generator. Biol. Cybernetics, 77, 367-380.

6 Bard Ermentrout, Type I membranes, phase resetting curves, and synchrony, Neural Computation, v.8 n.5, p.979-1001, July 1996